
Identification and evaluation of teaching practices that enhance numeracy achievement

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Note

This paper was prepared in 1998/1999 and some information in the paper may not reflect more recent developments

Disclaimer

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An overarching policy framework

In March 1997 all Australian Education Ministers agreed at a meeting of the Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA) to a new national goal for literacy and numeracy:

That every child leaving primary school should be numerate and be able to read, write and spell at an appropriate level.

In addition, Ministers adopted a new sub-goal:

That every child commencing school from 1998 will achieve a minimum acceptable literacy and numeracy standard within four years.

The *National Literacy and Numeracy Plan* agreed by Commonwealth, State and Territory Ministers of Education includes some key elements which aim to implement this goal and its sub-goal. Two key emphases of the national plan are the early identification of students' learning problems and tailoring of programs to meet individual needs. As a result, programs have been or are being developed by all States and Territories with a very strong focus on initiatives in literacy and numeracy related to the early years of schooling. Key components of these programs include teacher professional development and development of early intervention programs to support student learning.

Brief national overview of State/Territory numeracy programs¹

The Australian Capital Territory

System level assessment in literacy and numeracy was introduced in Year 5 in 1998, with plans to extend testing to Years 3, 7 and 9 in 1999. Reporting of achievement will be at system, school and individual level. Numeracy networks and professional development programs are under way in the ACT.

¹ This paper was prepared in 1998/1999 and some information in the paper may not reflect more recent developments

Tasmania

Through publications, such as *Numerate Students – Numerate Adults*, 1995 (Department of Education and the Arts), Tasmania has emphasised cross curriculum responsibilities and connections across the Key Learning Areas as essential for advancing students' overall development in numeracy. This initiative was the first in any Australian state to draw attention to the fundamental importance of numeracy for *all* students in this way. Its focus is on the development of numeracy at *all* stages of schooling, and in *all* areas of the curriculum (not just mathematics).

In the Tasmanian program, numeracy is focussed on the intelligent application of mathematical knowledge (knowledge of number, space and shape, measurement, calculation, and chance and data) to other school studies and to a wide range of practical contexts in everyday life. Numeracy is intended, therefore, to build on what students have already learned in these areas of mathematics. While teachers of mathematics have a major responsibility in this, a consistent and connected approach needs to be fostered across subject areas and among teachers. Students cannot be expected to make these connections unaided. Schools are provided with *Key Intended Numeracy Outcomes* (KINOs) at Years 2, 5 and 8 to guide teaching and assessment. These assessments also convey a powerful message to parents.

There is an additional Commonwealth-funded and state administered program for at risk students in the early secondary years. *Planning and teaching for Numeracy in Years 7 to 9* focusses on classroom structures and management, goal setting for 'at-risk' students and identifies a range of suitable teaching materials including software.

Flying Start is an established program in early literacy, numeracy and social skills — a local program being adapted from *Count Me In Too* (see NSW below) is a key aspect of its numeracy component.

Queensland

Supporting Literacy and Numeracy in Queensland Schools is a Commonwealth-funded, intersystemic project which is a joint initiative of Education Queensland, the Queensland Catholic Education Commission and the Association of Independent Schools of Queensland. It commenced in January 1998 and is due to be completed in December 1999. Currently, schools are required to use the Year 2 Diagnostic Net and complementary early intervention materials. Materials are also being produced to support teachers and schools in Years 1 to 5. Among these materials are: *Support a Maths Learner in Number and Space, Measurement and Data Common Learning Sequences* relating Years 3 to 5. These will be complemented by a multi-media professional

development package to support teachers in preparing programs for individuals and groups of children.

Victoria

Teacher professional development is a focus of the State-wide *Early Years Numeracy Program*. The principal components of the program are a structured classroom program, provision of additional assistance to meet students' identified needs, parent participation and teacher professional development. An *Early Numeracy Research Project* to be carried out over three years has been commissioned by the Department of Education. It aims to identify elements of mathematics classroom teaching that promote long-term learning and the acquisition of numeracy skills. It will investigate the effects of additional assistance, parent participation and school leadership on student achievement. The project will support more effective teaching practice through advice on classroom structure, and assessment embedded in instruction, within a framework of developmental stages, especially in the learning of early number.

Western Australia

The State's *First Steps Program* aims to improve mathematics learning in the lower and upper primary school years, particularly of those students who are judged to be at risk of not achieving their full potential. The *Transition Numeracy Project* is a Commonwealth-funded state administered program connecting the *First Steps Program* to work in the early years of secondary school through professional development focussing on identification of student needs and intervention strategies.

South Australia

The State's major initiative is an *Early Years of Schooling* project in support of the *National Literacy and Numeracy Professional Development Program*. This cross-sectoral program will 'provide assistance to teachers of reception to Year 5 to extend their repertoire of assessment techniques, to adapt teaching strategies to be more responsive to students' identified needs and abilities, and to focus on links between assessment and teaching'. It is being accompanied by the development of a state-wide assessment program in numeracy for children in their first term at school. Other programs include a Year 3 to 10 research project which focusses on 'action research' and a special program to 'investigate the contextual teaching and learning of mathematics in relation to the needs of Aboriginal students in a range of school settings across the State'.

Northern Territory

In the Northern Territory there is a strong focus on numeracy in the early years of schooling through school entry assessment in numeracy, teacher professional development and programs to involve parents and local communities. This is intended to lead to a system-wide strategy for early numeracy assessment and intervention for the year 2000. Began in 1998, the Territory has a *Numeracy in Schools* project directed at Indigenous students in primary and junior secondary years. There is currently a literacy and numeracy intervention pilot program for lower secondary school students in 15 schools.

New South Wales

The *Count Me In Too* early numeracy project operated in nearly 300 schools in 1998. Its aim is to improve outcomes of instruction in the early years of school (K–3) by providing teachers with support in using ‘learning frameworks’ to assess students’ strategies in counting and number, and to guide instruction. This project provides ‘first wave’ support for early numeracy by involving teachers in school based training and development in understanding the relative sophistication of students solution strategies for relatively simple number problems. It is being supported by a Commonwealth-funded cross-sectoral program, focussing on professional support for teachers in the use of assessment strategies in the first year of school, in intervention strategies to improve numeracy performance and to meet national numeracy benchmarks in numeracy. A Commonwealth and state funded program has also commenced for at-risk students in departmental schools in the lower secondary years. Entitled *Numeracy Plus*, it focusses on students’ thinking strategies in solving numerical tasks, teacher professional development and support for students in regular classroom settings.

Implications for policy in Australian schools

Among State and Territory programs which address numeracy in the early years of schooling, common threads are a clear focus on early number and counting strategies and application of recent research in these areas. Key elements of such research will be discussed later in this report in reference to specific numeracy intervention programs where there is a strong emphasis on teachers using information about children’s understanding, based on well designed assessment tasks in planning an instructional program.

Implementation of numeracy programs is beginning to be linked to systemic assessment of student achievement and is proposed to be linked to National

Benchmarks in numeracy across all States and Territories. In Tasmania, for example, Year 9 (1997) and Years 3 and 7 (1998) have already been involved in state-wide numeracy testing. From 1998, all K–8 students are to receive an annual numeracy report in addition to a regular report in Mathematics. From 2000, Tasmanian students in Years 3, 5, 7, and 9 will be tested for numeracy every two years. In Victoria, testing is already established through the *Learning Assessment Program* (LAP) in Years 3 and 5. In 1999, the *Victorian Student Achievement Monitor* (VSAM) will be introduced in Years 7 and 9. Related developments are occurring elsewhere in Australia and further developments are anticipated.

Among the various State and Territory programs, there is an implicit recognition of changing emphases within numeracy as students move through primary school. These changing emphases in how ‘numeracy’ is interpreted at different stages of schooling will be discussed later in this report. Numeracy intervention programs in the early years of school appear to have a much firmer foundation in relevant research. Numeracy programs in the later years of primary school and in secondary school appear to have a much thinner base of *applied* research to support them. This is not to say that little research has been carried out in these areas. Extensive research-based ideas for mathematics teaching in the middle grades of primary school have been collected by Owens (1993); and extensive research in the learning of geometrical concepts has been carried out by van Hiele (1986) and Pegg (1992), and in fractions and proportionality by Thompson and Thompson (1996), to mention just a few among many academic researchers into children’s mathematical understanding. But academic or pure research falls a long way short of applying research findings to the school curriculum in ways that would support numeracy programs. Research findings are not directly applicable to the classroom without considerable work *to link research-based knowledge to assessment and to the design of instructional activities*.

For some students, by the end of primary school, negative affective factors clearly combine with poor learning to make the goal of intervention and support much more difficult to achieve. These negative factors are equally worrying as such students move into secondary school. A Year 7 *Mathematics Recovery Program* at Bathurst (Thomas and Donaldson, 1995) has contrasted different approaches to organising Year 7 Mathematics classes when attempting to provide for the needs of less able students. School-based research is needed into how school organisational arrangements at the start of secondary schooling can best provide for the needs of all students. There is an urgent need to provide exemplars of good teaching, of assessment strategies, and of classroom structures to support those students who are not achieving by the time they enter secondary school. These problems become even more difficult in the final years of secondary school, with many of those remaining at school being disaffected students.

In the United States, an attempt has been made to apply research from The Netherlands, namely from the Freudenthal Institute in Utrecht, in developing a whole curriculum for grades 5 to 8 (Romberg, Allison, Clarke, Clarke, Pedro, & Spence, 1993). The project, *Mathematics in Context*, is a collaboration between the Freudenthal Institute and the Wisconsin Center for Educational Research. The project produced thirty three units of work, built around problem-solving contexts. Early research on implementation is promising in terms of student achievement and affective components.

All these areas need to be connected to related policy initiatives and studies overseas. Several of these will be discussed in detail in subsequent sections of this paper, notably the UK Final Report, *The Implementation of the National Numeracy Strategy* (England 1998), and the study: *Effective Teachers of Numeracy — Final Report* of a study carried out for the Teacher Training Agency 1995–96 by the School of Education, (Askew, Brown, Rhodes, Johnson and William, 1997, King's College, University of London).

Defining numeracy

The following definition taken from *The Implementation of the National Numeracy Strategy* (1998) gives a well-rounded description of numeracy. The first elements of this definition clearly apply to the early years of primary school, but its emphasis progressively shifts to the middle and upper primary school years (and applies to the early secondary years as well). According to the *Final Report, The Implementation of the National Numeracy Strategy* (Reynolds et al., 1998), numerate primary pupils should:

- have a sense of the size of a number and where it fits into the number system;
- know by heart number facts such as number bonds, multiplication tables, division facts, doubles and halves;
- use what they know by heart to figure out answers mentally;
- calculate accurately and efficiently, both mentally and on paper, drawing on a range of calculation strategies;
- recognise when it is appropriate to use a calculator — and when it is not — and be able to use one effectively;
- make sense of number problems, including non-routine problems, and recognise the operations needed to solve them;
- explain their methods and reasoning using correct mathematical terms; judge whether their answers are reasonable; and have strategies for checking them where necessary;

- suggest suitable units for measuring, and make sensible estimates of measurements; and
- explain and make predictions from the numbers in graphs, diagrams, charts and tables (para. 15).

By the end of the compulsory years of secondary school, all of the above expectations apply. By that stage, numerate students should be confident and competent in all nine areas. In the final stage of schooling, students need to demonstrate much more clearly the capacity to apply their mathematical understanding to a range of practical contexts relating to their other studies, personal work and to familiar daily contexts. This shift in emphasis is captured in the opening lines of the *Policy on Numeracy Education in Schools* published by the Australian Association of Mathematics Teachers (1998). The AAMT policy statement commences with the statement that:

To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.

The emphasis is perhaps inadvertent, since the policy statement aims to address all years of schooling. To be fair, the AAMT statement then goes on to underline the fundamental importance of numeracy in school education.

As students near the end of schooling, the focus of numeracy changes. The expectation is that all students, especially those who are not proceeding with tertiary oriented courses in mathematics, will be provided with courses which support their successful transition from school to work and further education and training. Such courses in mathematics should provide for the continuing mathematical development of students entering the last phase of schooling and should strengthen students' mathematical skills needed to support other school subjects and possibly Vocational Education and Training (VET) studies. Courses for these students need a strong emphasis on using mathematics in practical contexts relating to personal work and study and should encourage appropriate use of technology in all areas of study. There is an urgent call for making links between what is studied in mathematics courses and other curriculum areas, students' work experience, and vocational aspirations and for making a continuing study of mathematics attractive to all students.

Shifting emphases in definitions of numeracy

In this paper, three quite clear transitions in the definition of numeracy are identified. In the early years of schools, the focus of numeracy programs tends to be directed to the development of number concepts and counting skills which are deemed to be essential to students' later mathematical development. Speaking of the middle primary years onward, *The Implementation of the National Numeracy Strategy* (1998)

defines numeracy in terms of competence in working with number and with applications of number throughout the school curriculum. In this middle phase, the focus is *primarily* about achievement at school. It might also be argued that the goal of having more numerate students can never be divorced from the broader personal, social and work-related contexts emphasised in the AAMT policy; contexts which become very important and explicit as students near the end of the compulsory years of school. It is important to note that there are not three different definitions. They clearly interlock and interweave. What is evident is that the focus of the definition does alter and takes on a different nuance depending on the age and stage of schooling.

Numeracy as a political and social term

Numeracy is first and foremost a political and social issue. It is significant that the push for a national numeracy strategy has not come from educational researchers. It has come from politicians, educators and industry leaders who sense the fundamental importance of some areas of mathematics for children's overall success at school, in particular, and for their capacity to take on further study, and to move successfully from school to further training or study, into paid work, and into a society where they need to deal confidently with numerical information and other forms of quantitative data. Because its driving forces are political and social, the definition of numeracy is not always precise. It is similar in nature to other educational slogans, such as calls to raise literacy standards, demands for drug education, or to introduce civics and citizenship education. These slogans are powerful ways of drawing attention to the work of schools, of shifting priorities, and of directing resources. The current focus on numeracy is an opportunity and a challenge to schools, school systems and to the wider Australian community.

The emergence of numeracy into the education agenda for Australian schools is not the result of research. It is not an issue which is owned or controlled by the mathematics education community or the research community, although their involvement will be required, very likely on terms which are determined outside these particular constituencies. It represents a shift in social and political consciousness. As such, it takes on some features of a slogan, just as a focus on literacy (or to put it more succinctly, 'What are the standards of literacy that society expects of its schools and from its citizens generally?') has been a slogan or rallying call for a much longer period of time. There may well be some debate about how a definition of literacy is influenced by the rapid developments in electronic communication, but the fundamental issue appears to be an ability to read and to comprehend meaning in these changing contexts. The current focus on standards of numeracy is both a call for schools to be clear about what to teach and what all

children should be expected to know and do, and an insistence that schools be able to give an account of their performance in this area.

Implications for policy in Australian schools

A national strategy for numeracy must address students' needs at different stages of schooling. A proper concentration on the development of children's number concepts and counting skills in the early years of school needs to be translated into a broader definition of numeracy during the middle and upper years of primary school. This same broad emphasis should continue into the early years of secondary school. There will, of course, be outcomes for mathematics learning, *other than those identified for numeracy*, which will need to be addressed at all levels of schooling. Starting in the primary school there should be a focus on practical applications of mathematics which matches students' increasing maturity and experience. A nationwide strategy is needed on numeracy in the 'middle years'. Such a strategy must bridge the gap between primary and secondary schooling where so many students seem to fall between the cracks. In the final years of secondary schooling, numeracy programs need to give increased focus — for some students an exclusive focus — on developing numeracy skills which equip young people to move from school to further study, work and training. For students who are taking tertiary preparation courses in mathematics, these important links are relatively easy to make, given the growing emphasis on applications of mathematics and mathematical modelling in higher level school mathematics courses.

Options for the classroom program

Models of instruction which rely on 'chalk and talk', drilling in rote procedures, or self-discovery, offer little hope for students who are experiencing difficulty in school mathematics.

One model of instruction which has clear relevance to Australian schools is drawn from the King's College *Effective Teachers of Numeracy* (1997) study and from the recommendations contained in the English report, *The Implementation of the National Numeracy Strategy* (1998). A similar model of instruction is reported by Stigler and Perry (1998) in their discussion of cross-cultural effects in the Third International Mathematics and Science Study (TIMSS). They report that teachers in many Asian classrooms, notably in Japan and Singapore, combine a blend of teacher directed instruction and small-group or individual work based on carefully constructed tasks to an extent that is much more evident than in comparable year-level classrooms in the USA.

Guiding students toward more effective procedures and toward more complex mathematical representations is also very clearly embodied in the program of Cognitively Guided Instruction (CGI) developed by Carpenter & Fennema (1992) and later evaluated by Carpenter, Fennema & Franke (1996) and by Carpenter, Franke, Jacobs, Fennema, & Empson (1997). CGI is a highly effective research-based program to assist teachers to identify students' standard and invented strategies in solving word problems involving single and double-digit numbers and related addition and subtraction operations. The research basis for CGI is founded in the work of Carpenter and Moser (1982) on children's early number understanding when applied to simple number problems. CGI is not an instructional program in the sense that teachers are provided with a ready made program or a script. Its emphasis is on helping teachers to identify the range of standard and invented strategies used by students to solve number problems, to know the types of problems that students typically find more difficult, and to ensure that their teaching program builds on what children know and takes them to a next level of mathematical sophistication.

In programs such as CGI, an essential part of the teachers' role is to know in advance what strategies are likely to be elicited in response to particular items, and to identify these responses as they move around the room observing students' work. Children are then invited to report on their group or individual work to the whole class. Those asked to report will have been carefully chosen by the teacher during the previous phases. Strategies are then discussed, compared and where necessary evaluated.

In these various models, teachers play a pivotal role in introducing key ideas for the lesson, and then set several tasks on which students are to work during the lesson. Tasks are carefully devised to elicit different correct strategies, as well as disclosing incorrect strategies. (Among the correct strategies, some may be more effective than others. A key decision for the teacher will be when to decide to assist those students who are ready to move on to more effective strategies, and when to allow others to continue with their own strategies because they are not yet ready to move on.) In these models, teachers are not expected to remain neutral, but to guide students towards effective procedures — not necessarily a single procedure — when they are judged capable of making this step. Research by Dominick (1991) and Kamii and Dominick (1998), shows that premature movement of students towards prescribed computational algorithms causes confusion for those students who have still to achieve a firm understanding of place-value. The counter side of this argument is that it is just as damaging to leave students to find their own way.

Implications for policy in Australian schools

The focus of any strategy for numeracy in Australian schools must be on supporting effective teaching and learning, primarily within the regular classroom. An effective

nation-wide strategy will require changes to those models of classroom instruction which have placed too much responsibility on children's self-directed learning through relatively unstructured activities or through activities on which there is no whole-class feedback or discussion. This should not be interpreted as supporting a return to direct instruction with limited teacher-student and student-student interaction, or to other models such as 'chalk and talk' or 'drill and practice'.

If one accepts that a teacher's role is to know in advance what strategies are likely to be elicited in response to particular tasks or questions, creating such a knowledge base has major implications for teacher preparation and for ongoing teacher professional development.

Evidence regarding general teaching strategies designed to enhance numeracy for all students

Effective Teachers of Numeracy — the King's College (1997) study

The study: *Effective Teachers of Numeracy — Final Report* which was carried out for the Teacher Training Agency 1995–96 by the School of Education, King's College, University of London (Askew, Brown, Rhodes, Johnson and Wiliam, 1997) identified effective teachers of numeracy in a range of schools by looking at mean test scores of students over time.

The aims of the study *Effective Teachers of Numeracy* (1997) were to identify:

- key factors which enable teachers to put effective teaching of numeracy into practice in the primary school phase; and
- strategies which would enable those factors to be more widely applied.

The working definition of numeracy used by the project was a broad one:

Numeracy is the ability to process, communicate and interpret numerical information in a variety of contexts.

Evidence was gathered from a sample of ninety teachers on what the teachers believed, knew, understood and did. Data were collected on over 2000 pupils on the resulting outcomes in terms of pupil learning gains.

These teachers and pupils came from eleven schools, selected from the primary schools in three local education authorities (some 587 schools in all), together with independent preparatory (private) schools in the same area. Eight of these eleven schools were selected because their mathematics teaching had already been shown to be highly effective; the other three schools in the sample included two 'average' and

one 'weak' in relation to mathematics. Judgements were made according to the available evidence about 'value-added' performance. (Data gathered in the course of the study suggested the lack of reliability of such classifications.)

A specially designed test of numeracy ('tiered' for different age ranges) was administered to the classes of these 90 teachers, first towards the beginning of the autumn term 1995, and again at the end of the spring term 1996. Average gains were calculated for each class, providing an indicator of 'teacher effectiveness' for the teachers in the sample. In order to broadly classify the relative gains, the teachers were grouped into three categories of *highly effective*, *effective* or *moderately effective*. This classification was made by putting the classes in rank order within year groups according to the average gains made.

From the sample of 90 teachers the research team worked more closely with 18 'case-study' teachers over two terms. A further 15 'validation' teachers were also observed and interviewed in order to check out hypotheses made on the basis of the case-study data and the whole sample.

Methods were selected and developed to provide data on teachers' beliefs, pedagogical and mathematical subject knowledge, professional development experiences, and practices.

Key conclusions of the study (see pp. 1–4 of the report) are as follows:

- i. *What distinguished highly effective teachers of numeracy from other teachers was a particular set of coherent beliefs and understandings which underpinned their teaching of numeracy. Their beliefs related to:*
 - *what it means to be numerate*
 - *the relationship between teaching and pupils' learning of numeracy*
 - *presentation and intervention strategies.*

The beliefs determined, for example, what type of questions teachers asked and how they followed them up, irrespective of whether they were talking to pupils individually, in a group or to the whole class.

- ii. *Highly effective teachers believed that being numerate requires:*
 - *having a rich network of connections between different mathematical ideas*
 - *being able to select and use strategies which are both efficient and effective.*

These teachers used corresponding teaching approaches which:

- connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words, symbols and diagrams;
- used pupils' descriptions of their methods and their reasoning to help establish connections and to address misconceptions;

- emphasised the importance of using mental, written, part-written or electronic methods of calculation which are the most efficient for the problem at hand; particularly emphasised the development of mental skills.
- iii. *Highly effective teachers believed in relation to pupils' learning that:*
- *almost all pupils are able to become numerate*
 - *pupils develop strategies and networks of ideas by being challenged to think through explaining, listening and problem solving.*

They used teaching approaches which:

- ensured that all pupils were being challenged and stretched, not just those who were able;
 - built upon pupils' own mental strategies for calculating, and helped them to become more efficient.
- iv. *Highly effective teachers believed, in relation to teaching that*
- *discussion of concepts and images is important in exemplifying the teacher's network of knowledge and skills, and in revealing pupils' thinking*
 - *it is the teacher's responsibility to intervene to assist the pupil to become more efficient in the use of calculating strategies.*

These teachers used approaches which:

- encouraged purposeful discussion in whole classes, small groups, or with individual pupils;
 - used a variety of different assessment and recording methods to monitor pupils' progress and to record their strategies for calculating, to inform planning and teaching.
- v. *Teachers who gave priority to pupils acquiring a collection of standard arithmetical methods over establishing understanding and connection produced lower numeracy gains.*

These teachers referred frequently to differences in pupils' ability to remember what was taught, and used teaching approaches which:

- dealt with areas of mathematics discretely;
 - emphasised teaching and practising standard methods in isolation and applying these to abstract or word problems without considering whether there were alternative, more efficient ways of solving a particular problem;
 - used assessment mainly as a check that taught methods had been learned, rather than as a means of informing subsequent teaching.
- vi. *Teachers who gave priority to the use of practical equipment rather than developing effective methods, and delayed the introduction of more abstract ideas until they felt a child was ready for them, also produced lower numeracy gains.*

These teachers used teaching approaches which:

- encouraged pupils to use practical equipment or any other method they were comfortable with;
 - dealt with areas of mathematics discretely, so as not to confuse the pupils.
- vii. *The teachers' beliefs and understandings of the mathematical and pedagogical purposes behind particular classroom practices seemed to be more important than the forms of practice themselves.*

For example, highly effective and moderately effective teachers in the study all sometimes used mental tests and used written exercises to practise skills. Whole class 'question-and-answer' teaching styles were used by both highly effective and comparatively less effective teachers, as were individualised and small group forms of organisation. Setting across an age group was used in schools with both high and low proportions of highly effective teachers. The same published mathematics schemes were used by highly effective and comparatively less effective teachers.

- viii. *Highly effective teachers of numeracy, themselves had knowledge and awareness of conceptual connections between the areas which they taught of the primary mathematics curriculum. In this study, being highly effective was not associated with having an A-level or degree in mathematics.*

Some, but not all, comparatively less effective teachers of numeracy, including some teachers with high mathematics qualifications, displayed knowledge that was:

- compartmentalised;
- framed in terms of standard procedures, without the underpinning of conceptual links.

For example, some teachers (who) were able to convert from a fraction to a decimal...(failed to consider)... when one should be used in preference to the other, or whether (the) two forms of representation are always equivalent.

- ix. *Highly effective teachers were much more likely than other teachers to have undertaken mathematics-specific continuing professional development over an extended period, and generally perceived this to be a significant factor in their development.*

Teachers described such courses as having led to major shifts in their thinking, achieved by discussion with other teachers and by talking to individual pupils in their own school as part of an assignment. These teachers displayed very positive attitudes to mathematics, in contrast to many of the teachers who had specialist mathematics qualifications.

- x. *In some schools, experienced and highly effective staff were able, over time, to assist other teachers to become more efficient through working closely with them in planning and evaluating detailed teaching approaches, and working together in the classroom.*

In most schools, there was a mixture of highly effective and less effective teachers of numeracy. Mathematics coordinators might themselves be highly effective but had insufficient time to significantly influence other teachers by working with them individually. In one school where stable staffing and resource allocation made this possible over a long period, there was evidence of significantly higher numeracy standards than in comparable schools, both in absolute and in value-added terms.

There appears to be no comparable studies of numeracy in other countries. However, supporting evidence for these findings is provided by several large curriculum development projects in the United States, funded by the National Science Foundation, which are intended to align with the National Council of Teachers of Mathematics (NCTM) Standards, and which emphasise problem solving, mathematical reasoning, connections, and communication.

The Connected Mathematics Project (CMP), based at Michigan State University, is one such project, written for sixth- to eighth-grade students and teachers. In an important study, data were collected for all three grade levels over a two year period, for both CMP and non-CMP classrooms (Zawojewski & Hoover, 1996). Three forms of assessment were used: the Iowa Test of Basic Skills, hour-long tests designed by the *Balanced Assessment Project*, and three 45-minute performance tasks. The importance of this choice of measurement instruments is that both 'basic skills' and problem solving capabilities could be assessed for both CMP and non-CMP students.

In summary, the findings were that sixth- and seventh-grade CMP students made healthy progress on traditional multiple-choice timed, standardised tests, similar to those of students in other curricula. Eighth-grade CMP students made greater gains on a traditional multiple-choice, timed standardised test compared to students in other curricula. Sixth-, seventh- and eighth-grade CMP students did better on open-response items which emphasised reasoning, communication, connections and problem solving, compared with students in other curricula. These findings are important because they show that the students who had been in the problem-solving program for three years outperformed other students on *both* problem solving *and* 'basic skills'. Other studies are beginning to emerge which paint a similar picture.

Implications for policy in Australian schools

Recent attempts to improve the teaching and learning of mathematics in Australia and in other countries such as England and the USA, have focussed on the development and endorsement by governments and educational authorities of standards for curriculum content and student performance. Important as these may be for establishing a shared vision of the work of schools, success will not occur merely by setting standards and by holding teachers and schools accountable for

meeting them. It is essential that research address the processes leading to success in classroom settings and in schools.

There have been no long term studies of numeracy in Australian schools which have the same depth and careful design as the King's College (1997) study. Australia has had very few large scale curriculum and teaching projects. A promising, recent development is the funding by the Victorian Ministry of Education of an *Early Numeracy Research Project*. To be conducted by the Monash and Australian Catholic Universities, one of the key objectives of the project is to evaluate the effect on student numeracy outcomes of key design elements of the State's early numeracy program and associated professional development activities. A range of instruments and processes will be used twice yearly to assess children's growth in numeracy in 25 trial and 25 reference schools. The analysis of these data will enable the identification of particularly effective teachers, coordinators and school communities in terms of children's numeracy learning. Detailed case studies will then enable elaboration of those teacher, coordinator and school community characteristics which enhance numeracy learning for wider applicability (Clarke, 1999). The results of this and other related studies should be disseminated nationally.

Several states and territories have conducted large scale programs for literacy in primary schools. Not only have these programs had the benefit of significant funding for a longer period of time, it is also clear that conceptual issues relating to literacy are better 'worked through' than for numeracy. There is a danger that discussion of a national strategy for numeracy will become 'bogged down' in a semantic dispute over definitions. While a degree of clarity is essential, it is already clear that one's conceptual unpacking of the term 'numeracy' shifts according to the stage of schooling one is discussing. Having established these features, it is important to move on and move the debate away from semantic issues to issues of practice.

It is crucial that lessons of effective professional development and classroom practice derived from literacy programs be applied to a national strategy for numeracy. This is necessary because the same schools and teachers will be involved. However, while having a clear eye for what has worked for literacy programs, there are several important differences. Many primary teachers, including many who are teaching in the early years, express considerable lack of confidence in their own knowledge and understanding of mathematics. There is also a need to improve perceptions of children regarding mathematics. Many children see mathematics as only 'sums', and by Year 1 some have begun to describe maths as 'boring'. There is a lack of shared understanding among teachers of the 'big ideas' of numeracy in the early years of schooling, combined with a lack of assessment instruments and processes which reflect these rich conceptions of school mathematics.

Special attention must be given to these differences in developing a national strategy for numeracy. All programs for numeracy in Australian schools must be informed by a broad understanding of pupils' growth in numeracy and knowledge of how this should be assessed. A rigorous design for evaluation of numeracy programs, and sharp focus on teacher professional development is required.

Findings of the National Numeracy Project (England)

The findings and recommendations of the King's College study are consistent with and are supported by the findings of the *National Numeracy Project* (NNP) which has been conducted in England since 1996. The NNP shares 'many key aims and methods of working with other projects begun at about the same time, aimed at helping primary schools to teach the National Curriculum for mathematics in a way that raises standards of numeracy' (Reynolds et al. 1998).

The framework for teaching advocated for schools participating in the NNP has received wide support in England. In their final report, *The Implementation of the National Numeracy Strategy* (1998), the authors refer to the importance of solid evidence of the effectiveness of the NNP framework if they were to recommend a national strategy based on its premises. They report that they have been 'very impressed by the evident gains in pupils' achievements that the NNP has so far brought about'. They refer to results of standardised achievement tests with written and mental components administered to pupils as their schools came into the project. Pupils' progress was subsequently measured after one year in the project and after two years when the school leaves the project. Based on testing of more than 23 000 pupils, the authors reported on significant overall gains in mean age standardised scores. In particular, they note that 'those who were achieving at the lowest level at the beginning of their school's involvement in the NNP were those who gained most (Reynolds et al., 1998).

The teaching framework advocated by the NNP and now advocated in the report, *The Implementation of the National Numeracy Strategy* (1998), is that all primary schools should provide a daily mathematics lesson of between 45 and 60 minutes for all children, with high quality teaching. In particular, the report notes that 'a higher proportion of these lessons should focus on developing numeracy skills' (para. 27), and a much higher proportion of time than was previously the case in mathematics should be spent teaching the whole class together.

These recommendations need to be treated carefully. They are certainly directed against a style of teaching where mathematics is *loosely* and at times *incoherently* integrated into the study of other subjects. This is not to underrate the importance of integrating mathematics in real contexts. It is simply to say that it requires very

knowledgeable teachers to make these links clear to students and to develop consistent and coherent threads of mathematical understanding.

There is a danger that the report's recommendations will be read as advocating a return to whole class instruction where the teacher directs the whole lesson from the front of the class. While the report does refer to 'direct teaching', this should be interpreted along lines similar to the King's College study, i.e. emphasising the primary responsibility of the teacher to plan instruction, to provide opportunities within the lesson for students to interact with other pupils and with the teacher, in regular and sustained ways that help them to make links and refine their understanding. These features are certainly not likely to be developed in a class where the teacher does most of the talking or where a teacher interprets 'interaction' as having pupils responding to disconnected drills and isolated routines.

The report outlines three principles which would support a broader and richer view of the teacher's role in the planning and delivery of daily lessons. 'Direct teaching' is taken to mean that the teacher:

- gives instruction and demonstrates, explains and illustrates mathematics, setting the work in *different* contexts and *linking it to previous work* (my emphasis);
- maximises opportunities to interact with pupils so that they can talk and *be listened to*, and *receive feedback* (my emphasis) that helps them to develop their mathematical knowledge, skills and understanding; and
- allows pupils to show what they *know*, *explain* their thinking and methods, and *suggest alternative* ways of tackling problems (para. 22) (my emphasis).

Clearly, this does not mean a focus on 'chalk and talk' with the teacher simply transmitting mathematical procedures to the class. Indeed, the authors of *The Implementation of the National Numeracy Strategy* (1998) explicitly advocate a style of teaching that commences with about 5 to 10 minutes where the teachers work with the whole class to review or rehearse key skills and knowledge. Then, the next 30 or 40 minutes are spent either in whole class, groups, pairs or individuals working on clearly designed tasks or problems, during which the teacher's role is to interact with students, identifying interesting approaches and misconceptions, and at times intervening. Then for the last 5 to 10 minutes the whole class is brought back into plenary mode to share insights and findings from the preceding stages of the lesson.

The authors caution that 'this outline should not be seen as a mechanistic recipe to be followed'. They point to the fundamental importance of teachers' using their professional judgement to decide about choice of activities, timing and organisation of each part of the lesson. They expect that there will be a varied mix of work with the whole class, groups, pairs and individuals at different times. But the authors return to the key importance of spending time working with the whole class. They

note that the report's recommendations, if adopted, will mean greater changes for some schools than for others.

Some cautionary comments

These recommendations may seem obvious but they are open to wide interpretation. For example, what is meant by 'summing up' may seem obvious. Yet research by Becker, Silver, Kantowski, Travers, & Wilson (1990) and by Fujii, Shimizu, Kumagai and Sugiyama (1998), clearly shows that summing up is interpreted quite differently in Japanese and US classrooms, for example. In both samples of schools studied, teachers spent a significant amount of time discussing and comparing students' proposed solutions in the main part of the lesson. In the US sample, the amount of time spent discussing and comparing students' proposed solutions was somewhat less (37% of total time) than the amount of time (46%) spent in the Japanese classrooms. In the latter classes, discussion tended to be whole-class rather than with individuals and small groups as was the case in the US classrooms.

As discussed by Stigler and Perry (1998), the culture of the classroom in many Asian countries appears to favour whole-class review and evaluation of methods, far more than in comparable classrooms in the USA. For example, these authors point to the practice in many Asian classrooms for teachers and students to spend a whole lesson working together through one carefully chosen problem and discussing various solution strategies used by students. In contrast, they report that in USA classrooms individual 'seat work' is more the norm — where students practice procedures that have been briefly taught at the start of the lesson. One suspects that the same is true in Australia, although there have been no comparable research studies based in Australian schools.

The Japanese researchers, however, saw the greatest differences in what US teachers called summing up compared to their Japanese counterparts. While US teachers did round off their lessons, there was, from the Japanese point of view, no real 'summing up' in terms of the teachers' attempts to tie together conceptually what the students had presented. There was also, they thought, a reticence to evaluate or to draw attention to the most promising lines of inquiry, and rarely any conceptual link or 'hook' to connect what was done today to tomorrow's lesson. In the opinion, of the Japanese researchers, their US colleagues tended to treat 'summing up' in purely procedural terms, i.e. 'packing up', setting homework, and drawing pupils' attention to what had been done that lesson, but most of the time 'sitting on the fence' when considering a range of different strategies presented by students.

A recent important development has been the decision by the Australian Research Council to award ARC Large Research Grant funding for a study entitled

Mathematics classrooms in four countries: the learners' perspective. This project seeks to build on the Third international Mathematics and Science Study (TIMSS) by studying mathematics classrooms in Japan, Germany, Australia and the USA from the perspective of the student. Whereas the TIMSS video project focussed exclusively on the actions and perspectives of teachers, this project goes one step further in claiming that improvement of mathematics learning must be founded upon an understanding of both teaching and learning, and the relation of both activities to student achievement.

In particular, the project will focus on processes which assist the calibration of teaching practice across different cultural settings and students' interpretations of mathematics classrooms. It will use student interviews, student test performances, and classroom actions and dialogue in order to identify learning outcomes that can be associated with the student's participation in each lesson and the sequence of lessons as a whole.

Implications for policy in Australian schools

Presenting a purely *structural* outline of classroom reform is all too likely to focus on surface appearances and procedural issues, without drawing attention to important features in the teaching of numeracy. Similar risks apply to those who take key features of early literacy based programs and seek to transport them to the teaching of numeracy — without fully appreciating the nature of what is being taught and how new topics in mathematics present particular challenges for children's learning. It is as though there is an assumption that general and universal laws of learning are embedded in literacy programs, and that these can be applied everywhere in the curriculum.

A nation-wide numeracy strategy cannot afford to focus solely on the early years of schooling. Certainly, getting things right in the early years appears to make very good sense. But in numeracy, having mastered early number and counting skills, there are still significant conceptual hurdles to be mastered in the middle years of schooling, for example, in decimal understanding, ratio, measurement and shape. Ensuring that these are well taught and well learnt — including learning to apply the concepts — is critical to ensuring effective numeracy development.

Use of calculators

The Los Angeles Times (December 11, 1998) carried a story about new guidelines for mathematics teaching issued by the Californian State Board of Education. This front-page article reported that the Board had recently adopted new guidelines for

mathematics instruction which discouraged use of calculators until children could demonstrate that they could carry out the four arithmetic operations on their own. The decision was said to be a reaction to poor scores by the State's students on national tests, and to what was seen as the proliferating use of calculators in primary school classrooms. Professor Hung-Hsi Wu, a mathematician from the University of California at Berkeley who had been responsible for editing the guidelines said that he did not want to 'discourage the judicious use of calculators, especially for advanced studies (in mathematics) in middle school and high school', but claimed that 'there is no hard data to prove that the benefits of calculators and much anecdotal evidence to suggest that students lean on the devices to conceal their defective knowledge'.

In the United Kingdom, the *Final Report, The Implementation of the National Numeracy Strategy* (Reynolds et al., 1998) recommends that 'calculators are best used in primary schools in the later years of Key Stage 3, (*in other words, their use should be limited in the early years*), and should not be used as a prop for simple arithmetic. Teachers should teach pupils how to use them constructively and efficiently' (Summary of Recommendations, no. 16).

Although the latter recommendation is far more cautious than that attributed to the California Board of Education, these recent moves certainly represent a retreat from the position advocated not long ago by Shuard, Walsh, Goodwin and Worcester (1991) in their book, *Calculators, children and mathematics*, written in connection with their *Calculator Aware Number* (CAN) curriculum project. There the authors argued that children should be allowed to use calculators *in the same way as adults use them: at their own choice, whenever they wish to* (my emphasis). The same authors also argued that the traditional (vertical) pencil and paper methods for the four operations not be taught, since students would always use calculators for operations which they could not do mentally. In a later report, Shuard (1992) suggested that children in the CAN project showed an interest 'in mental calculation and other methods of non-calculator calculation. They want to be able to 'do it themselves' rather than rely on the calculator' (p. 36).

The role of calculators in the early and subsequent years of primary school is a contentious issue internationally, and in this country there is a clear need for advice regarding their appropriate use. If mental computation, a developing number sense and a growing understanding of the number system are among the most important goals of these years, as evidenced in the curriculum documents of all States and Territories, then there is a strong case for some restricted access to calculators and for clear guidance to teachers about when they should be used. The same point is echoed in the Summary of Recommendations in the *Final Report, The Implementation of the National Numeracy Strategy*, quoted above, by Reynolds et al. (1998) where they also encourage a greater emphasis on oral and mental calculation in the teaching of

mathematics in Key Stage 1 and 2 (Summary of Recommendations, no. 15). Recommendations 15 and 16 of the report both point to the need for clearer guidance to teachers about when written calculation methods should be introduced, and by implication, on the proper and effective use of calculators in these early years of school. Several points need to be made in support of this position.

Firstly, children in the early years of primary schools and later, need to be helped to make sensible decisions about whether mental computation, pencil and paper or a calculator approach is required. This does not happen unless the teacher is clear about which approach is best suited to particular tasks and to particular stages of schooling.

Secondly, use of calculators can support the development of important number concepts. Swan and Sparrow (1998) recommend use of the constant function, or repeated addition in order to disclose number patterns. Clearly, this needs careful guidance by the teacher. For example, adding, subtracting, multiplying and dividing by tens can be supported through use of a calculator. Children need to be alerted to the patterns and build these patterns into their knowledge of the number system. However, once these patterns have been established, it is essential for children to know that using a calculator is an inefficient way of adding, subtracting, multiplying and dividing by ten. At a later stages, mental methods should be preferred for the four operations involving one hundred, one thousand, and so on, once these patterns have been explored through use of a calculator. Groves and Stacey (1998) make a similar point that:

At some stage during elementary school, children should become adept at dividing by ten and its powers. We would not wish them to be dependent on a calculator for this simple computation. However, it is essential that they do not merely 'shift' the decimal points around but know what the process involves and what their answers mean (p. 126).

Similarly, one would argue that students and adults should be able to compute mentally the result of dividing \$59.00 by 10, or any similar problem. However, using a calculator is appropriate when dividing such an amount by 3 or 7 if an exact result is required.

Thirdly, children and adults need to have a sense of what the answer should be in case they slip up using the keys of the calculator. For example, they need to know that when they use a calculator to divide \$59 by 3 the answer will be 'close to and a bit less than \$20', since that would be the result of dividing \$60 by 3. Swan and Sparrow (1998) also argue that access to calculators allows children to work with large numbers and to see patterns involving large numbers, long before this would be possible using pencil and paper methods. In the CAN project, children were frequently encouraged to select their own starting number when exploring patterns,

'allowing the more able to extend their command of numbers while the less able remained within their own limits' (Shuard, 1992, p. 39).

Research evidence relating to the effects of calculator use is substantial but not overwhelming. Several research studies by Hembree and Dessart (1986), Shuard (1992) and Groves and Stacey (1998) appear to show that there are no detrimental effects on children's learning of number concepts and skills as a result of long term use of calculators. Shuard's (1992) study involved children who had been using calculators from age six. Groves and Stacey (1998) report on an extensive program of testing and interviewing slightly older students who participated in the *Calculators and Primary Mathematics* project:

The performance of third- and fourth-grade children... was compared with that of a control group consisting of the last cohort of third- and fourth-grade children at the same schools who had not been part of the project. The interviews showed that children with long-term experience with calculators performed better overall than children without such experience — both on questions for which they could use any tool of their choice and on mental computation....They also made more appropriate choices of calculating devices... and were better able to interpret their answers when using a calculator especially when decimal answers were involved....For further details of the results from these tests and interviews see Groves (1993, 1994) and Stacey (1994) (p. 128).

In reviewing these reports it is essential to realise that children and teachers involved in these calculator-based projects were operating within settings supported by prepared curriculum materials, associated teacher development activities and regular contact with mathematics education researchers. It would be unfair to compare their use of calculators, fashioned within a framework provided by project materials and associated teacher development, with that of teachers who simply allow students to use calculators whenever they want to. These research studies, strongly confirm the need for school systems and educational authorities to provide a clear framework for all schools in the use of calculators. Such a framework needs to affirm the importance of developing number sense, and of teaching students how to select sensibly and efficiently between mental, written and calculator-based methods in classroom work and in assessments.

Implications for policy in Australian schools

There is a clear need for advice regarding the appropriate use of calculators in the early years of school, and in subsequent years. If mental computation, a developing number sense and a growing understanding of the number system are among the most important goals of these years, then it is essential that calculator use take place in well constructed and well thought out programs, and that clear guidance and support be provided to teachers about sensible and appropriate use of calculators.

Such attention to issues of calculator use will need to be integrated into the initiatives and resourcing of schools in the broadest sense.

Resources for schools

While praising the encouraging results from preceding work on the U.K. National Numeracy Project (NNP), the authors of the *Final Report, The Implementation of the National Numeracy Strategy* (Reynolds et al., 1998) note that it will not be possible, 'within available financial and human resources, to give every primary and special school in the country the same level of support that schools in the NNP have received' (para. 18). It would be a great pity if schools were persuaded that substantial gains in numeracy could be achieved simply by a return to 'direct teaching' without accompanying teacher development to ensure that teachers are confident in their own understanding of mathematics and in their ability to provide rich and varied opportunities for children to develop mathematical skills and understanding. The qualities of effective teaching identified in the King's College study do not occur without hard work. They require investment of resources in teacher development. They also require support from colleagues, from schools systems, and a sound initial teacher training. Australian schools, too, need a major investment in teacher professional development, supported by well coordinated national initiatives, and ongoing research into what programs work and why they are effective.

Implications for policy in Australian schools

A nationally coordinated strategy on numeracy needs to build on the strengths of programs already under way in the various States and Territories. However, there is clearly a significant role for national coordination to ensure that local programs are well informed by best practice in other Australian States and by relevant overseas and local research. There needs to be a national clearing house for information about numeracy programs and projects. Dissemination of research information and information about overseas projects is another important area that is better handled nationally. Finally, across the various State and Territory programs there is an urgent need for rigorous and well designed research, similar to that undertaken in England, and anticipated in the Victorian *Early Numeracy Research Project*, to evaluate programs and to identify what works.

Programs and strategies designed to assist students who are

identified as 'at risk'

Traditional approaches to assisting students with learning difficulties in numeracy have tended to focus on remediation and withdrawal. Typically these approaches involved withdrawal from the regular classroom of children in the middle and upper years of primary school, thus removing the classroom teacher from the cycle of intervention and support. This focus on withdrawal undervalues the importance of classroom instruction in detecting and overcoming difficulties, and the support of classroom peers. It is pleasing to note that some recently developed programs in this area have a focus on the early years of schooling and are able to complement and integrate with regular classroom teaching.

There is now an extensive and well-documented literature in Australia on mathematics intervention strategies such as *Mathematics Recovery* (Wright, in press; Wright, Stanger, Cowper and Dyson, 1996; Wright, 1994; Wright, 1991), *Count Me in Too* (Mulligan and Bobis, 1999) or *Mathematics Intervention* (Pearn and Merrifield, 1998). Australian researchers and teachers can rightly claim to have provided international leadership in these areas. The theory and methods of *Mathematics Recovery*, for example, have been drawn from the extensive research into children's early number thinking by Steffe and colleagues (Steffe, 1992; Steffe and Cobb, 1988; Steffe, von Glaserfeld, Richards and Cobb, 1983) and from related research by Fuson, Richards, Briars (1982); and by Carpenter and Moser, (1982).

Early intervention programs in mathematics, like those in early reading, such as *Reading Recovery* (Clay, 1987), involve the application of developmental models of children's thinking. Research into children's early thinking in arithmetic has been applied successfully to classroom programs as well as to programs of early intervention. In the area of early number and counting, the most widely applied model involves five stages of early arithmetic learning. Like all developmental models, early intervention programs assume that unless children successfully master a given stage, their progress to and through the next stage becomes problematic. It needs to be pointed out that the five stages are not narrowly specified behavioural goals.

The five stages have been described by Steffe, von Glaserfeld, Richards and Cobb (1983) and by Steffe and Cobb (1988) as follows:

- *Perceptual*. Students are limited to counting items they perceive.
- *Figurative*. Students can solve problems involving concealed items, but in doing so count from the starting number — one — when solving addition problems. For example, if required to add two collections, one of six items and one of three items both of which are concealed, students must first count the six items, then count the three items by continuing their count from seven to nine. In dealing with simple subtraction problems, children

need at this stage to represent the problem using materials and to enact the subtraction as an actual 'taking away'.

- *Initial number sequence.* Students are able to 'count on' to solve addition problems and missing addend problems involving concealed items. In the previous problem, they would typically start counting at six and count on three more to get nine. Students may also be able to use a count-down-from strategy. For example, children at this stage do not need to use materials to solve the problem 14 subtract 3. They will typically say '14, 13, 12, 11' and give the answer as 11.
- *Implicitly nested number sequence.* Students can count-on-from and count-down-from, choosing the most effective strategy to solve problems. They generally count down or backwards in order to solve subtraction problems. For example, for the task 14 subtract what leaves 11, children at this stage might say '13, 12, 11, so 14 subtract 3 leaves 11' and do not need to represent the problem using materials.
- *Explicitly nested number sequence.* Students are simultaneously aware of two number sequences and can disembed smaller composite units from the composite unit that contains it. They understand that addition and subtraction are inverse operations. For example, when solving $12 - 9$, students can count on from 9 giving the answer 'three'; recognising that the problem can be represented as 'the difference between' 12 and 9, and can also be solved by counting back from 12 to 9, rather than as a physical 'taking away' which is in fact more complicated and exposed to error. This stage is characterised by the emergence of strategies other than counting by ones. In the preceding example, a student has used a known addition fact, $9 + 3 = 12$ to solve $12 - 9$. Students might use a known addition fact, for example $6 + 6 = 12$ to solve $5 + 7$.

These five stages very clearly outline increasingly sophisticated cognitive development, and make very clear links between children's development of number knowledge and their ability to use a rich language to express their number thinking in increasingly subtle and sophisticated ways. Unless children are able to make these connections, their understanding of number and their ability to work successfully with numbers and their applications are at risk.

Early intervention programs such as *Mathematics Recovery*, which is organisationally similar to *Reading Recovery Program* (Clay, 1987), and related programs described by Pearn and Merrifield (1998) all focus on identifying children who, after one year of school, are unable to perform on a range of tasks involving counting strategies and number thinking. Identification is undertaken by a trained interviewer in a one-to-one interview. Children are either withdrawn from the regular classroom for one-to-one teaching in the case of *Mathematics Recovery*, or placed in small groups in the case of mathematics intervention programs described by Pearn and Merrifield (1998). Diagnostic tasks and instructional tasks are very well described. Children are

typically involved in such programs for several weeks. Wright (1998, personal communication) also reports that several schools have successfully adapted *Mathematics Recovery* to work with small groups of at-risk students at first, second and third grade levels.

Research reported for both programs shows significant gains among almost all of those involved. A few children still fail to make significant progress despite the very clearly focussed mathematical instruction and the benefits of one-to-one or small-group teaching. Wright (1996, p. 69) reports that many children, 'with no more than regular classroom instruction', made similar but usually less substantial gains than those involved in a *Mathematics Recovery* program'. However, he cautions that 'these advancements (of children with no more than regular classroom instruction) are *unlikely* (my emphasis) to be sufficient to bridge the ever widening gap between the least advanced and the average learners' (Wright, in press).

Given the resource intensive nature of intervention programs such as *Mathematics Recovery*, the reported gains made by participating children may not be unexpected. However, it is easy to move from this position to argue that the children in a regular classroom are likely to remain at risk *unless* they are withdrawn from the regular classroom and immersed in an intervention program. This issue is less clear. Wright himself does not make this claim, but it is a tempting generalisation for those who support intervention and withdrawal. In a *Mathematics Recovery* program, children benefit from one-to-one instruction. They also receive help from a teacher who has been specially trained to appreciate the cognitive linkages and connections which underpin children's counting skills and number thinking. They also benefit from having a teacher who is able to apply this knowledge in a very focussed program of instruction.

Wright (1998), in a personal communication, raises a key question, namely whether, in the lower primary years, a significant percentage of children need more than a good classroom program. There are yet, as he points out, no research studies which chart the progress of at-risk students who undertake a classroom program which has the goal of addressing their low-attainment. Such studies are urgently needed to compare the effectiveness of classroom-based programs to resource-intensive withdrawal programs such as *Mathematics Recovery*.

Could a teacher in the regular classroom who had similar training bring about some of the same gains? That question is critical, given that the goal of intervention programs, such as *Mathematics Recovery*, is for children to return to the regular classroom. It is a key question for teacher professional development given that the focus of both programs is children's early counting and number development. It is at the heart of the NSW early primary program, *Count me in too* (Mulligan and Bobis, 1999; Stewart, Wright and Gould, 1998; Wright, 1998) because the latter involves

application of the theory and methods of *Mathematics Recovery* to whole class teaching and learning. For example, teachers use videotape to record assessments of individual students in order to enhance their understanding of children's early number thinking. These recordings are a vital part of teachers' professional development.

Count Me in Too is a whole class program, and is not designed to identify particular student weaknesses according to any preconceived timetable. It does not correspond, either in intention or practice, to programs such as *Mathematics Recovery* as discussed in this report. In a classroom where CMIT is operating, all children are involved in activities designed to support their movement to the next level of the framework, however long this might take. Children are assessed in relation to their individual progress rather than in reference to any specific time-frame.

Since intervention-and-withdrawal programs are resource intensive and, therefore, costly in terms of the limited resources available to schools, we need to look carefully at programs such as *Count Me in Too* in order to support the professional development of teachers and to enrich the quality of mathematical instruction in regular classrooms. The apparent success of *Count Me in Too* shows that 'regular' classroom teachers can use the learning framework implicit in these intervention programs, especially in the sophisticated connections they make in children's mathematical knowledge, starting from a detailed examination of where individual children are at, and then utilising well-designed instructional activities to support children's learning. Approaches to professional development used by *Mathematics Recovery* and *Count Me in Too* should be considered in programs in the upper primary and early secondary school years. An intervention program on decimals in the upper primary school (Stacey and Steinle, 1998; Steinle and Stacey, 1998) has produced significant gains in children's understanding while operating in a regular class room environment. This program deserves careful examination, as does work currently under way by Mulligan and others on the learning of multiplication in the middle primary years.

Implications for policy in Australian schools

These intervention programs all project similar messages which are critically important in informing teacher professional development in the area of numeracy.

These programs are all successful because they embody:

- a research-based view of children's mathematical thinking, which emphasises connections and linkages, rather than knowledge of discrete and isolated facts;
- use of carefully designed and 'rich' assessment tasks to guide instruction;

- well-developed instructional activities linked to these assessment tasks;
- a focus on the language of mathematics, in both spoken and written forms, as a powerful means of articulating, connecting and refining their mathematical thinking;
- opportunities for teachers to assess children's knowledge through structured interviews and through use of videotaped recording.

These points are strongly supported by the King's College study. Building connections and linkages in children's mathematical thinking is at the heart of any effective numeracy program. Assisting children to make these connections requires very careful and well prepared teaching. It certainly requires teachers to have a good sense of where students are coming from and what they can do. The use of rich assessment tasks to guide teachers in planning instruction is fundamental. Models of assessment currently endorsed by school systems in Australia have tended to focus on the use of large scale multi-item tests, such as Basic Skills Testing in NSW or the *Learning Assessment Program* (LAP) in Victoria. States and Territories have invested very considerable resources in such forms of testing which provide very strong messages to parents and to the wider community about the importance of literacy and numeracy, and help to define what these terms mean.

It is pleasing to note that these tests have adopted a relatively broad definition of numeracy in line with State and Territory curriculum statements, and they are to be used to gather evidence of achievement on nationally adopted 'benchmarks' of achievement in literacy and numeracy. Research by Clements and Ellerton (1995) provides some grounds for being cautious on the use of responses from large-scale testing for describing students' mathematical understanding. Students were interviewed on their attempts at sixteen multiple-choice and short-answer items. In their research, based on interviews with sixty-five Year 8 students from New South Wales, Clements and Ellerton (1995) suggest that:

about one-quarter of students' responses could be classified as either: (a) correct answers given by students who did not have a sound understanding of the mathematical knowledge, skills, concepts and principles which the questions were intended to 'cover'; or (b) incorrect answers given by students who had partial or full understanding' (p. 184).

Clearly more research is needed on this issue, but such data support the case for more intensive interviews with students in order to describe what students really understand; and to raise questions about whether one can safely infer from performance data on multiple-choice questions what students can explain and apply in more complex situations.

These forms of large scale testing can be expected to continue. There is a good case for more intensive testing of particular aspects of student understanding through

sampling such as through the continuing *Monitoring Standards in Education Program* in Western Australia. There is also a strong case for large scale testing to be complemented by a range of carefully designed assessment tasks which are better able to be used by teachers to explore children's thinking and to guide instructional planning. Resources therefore need to be spent on expanding the range of these rich assessment tasks which create direct links to teaching (see Beesey, Clarke, Clarke, Stephens and Sullivan, 1998). Finally, the importance of developing a rich and varied mathematical language is fundamental to any numeracy program. Connections and links in children's understanding require a very clear focus on the use of mathematical language in the classroom, and on opportunities being provided in the regular classroom for children to use and refine their own mathematical language.

Ways in which school-home links can be supported

Support from parents and the wider community will be essential if standards of numeracy are to be raised. However, while it is clear that literacy skills are widely valued and generally well understood by the community at large, it is not clear that numeracy enjoys the same degree of support; nor is it clear that parents and the community share a broadly based view of numeracy which has been discussed in this paper. It is also clear that while parents tend to be more confident in how they can and should support their children's growth in reading and literacy, many parents are far less clear about how they can or should support their children's development of numeracy skills. Some parents have a low degree of confidence in their own numeracy skills, and for this reason may tend to excuse difficulties which their children experience in numeracy which they would certainly not accept in reading or writing. Many parents may not see that discussion of everyday activities with their children, such as planning a trip, following a cooking recipe, or working out a holiday budget can all be utilised to improve children's understanding of and confidence in dealing with applications of mathematics.

The *Family Maths Program* in Australia developed a range of models for involving parents in school mathematics. These models, derived from and affiliated with the *Family Math Project* based at the Lawrence Hall of Science, Berkeley, California, could well be harnessed towards helping parents assist their children's numeracy development. Parents need to be made comfortable with the methods that are used in a school's mathematics program and recognise the power of the approaches as used by young children.

Marj Horne (1998) outlines a range of strategies by which parents can be directly involved in supporting children's numeracy. First, there is parental involvement on curriculum committees through official school organisations. This is clearly a role

open to very few parents. Second, there is now a well established practice in many primary schools of having parents assist in classrooms during mathematics lessons. Third, schools also involve parents in preparing materials to be used in classrooms and in other learning contexts, such as Mathematics Task Centres. Fourth, parents can be invited to participate with their children in a mathematics activity day, for example in exploring a mathematics trail together. Some schools also include mathematics puzzles and problems in their regular school newsletters and invite parents and children to work together in preparing responses and solutions. Fifth, parents can be involved in a more extended program with their children such as the *Family Mathematics Program*. Sixth, parents are frequently involved in supervising children's regular homework or discussing with their children what mathematics they have learned. Some schools provide advice to parents, for example on the school's web page, on how they can assist with homework or other assigned work in mathematics. Seventh, schools may also provide specially prepared activities for children and parents to work together on, such as completing a family tree, or preparing a map of a family's living space. Finally, parents are often involved directly in seeking assistance for their children in mathematics, outside the school structure, through tutors and coaching programs.

In pursuing several of these strategies, advice to parents is provided in the brochure *A Guide for Parents: Helping Your Children with Mathematics* prepared by the Mathematics Teaching and Learning Centre of the Australian Catholic University (1996). Advice is offered on everyday, family activities which parents can do with children, to support the mathematics learning of their children. The focus is in strengthening parental attitudes towards mathematics and in encouraging their children to see purpose and relevance in mathematics itself. These programs are not about making parents instructors in the narrow sense of that term, but rather motivators and encouragers. Behind all these strategies is an assumption that

parents' attitudes towards mathematics... do have an influence on their children's self concept, attitudes, expectations and hence achievement. The link in the chain of belief from parent attitude to student attitude seems to have some justification. Another link is a causal one between student attitude and student achievement. The nature of this link is not so clear, however, and needs further study (Horne, 1998, p. 124).

In her evaluation study of the *Family Mathematics Project* (FAMPA) Horne (1993) reports small positive effects on several measures as a result of participation in a FAMPA program. These results are consistent with her analysis of studies of other family mathematics programs. Participation in such programs is significant in changing some parents' attitudes to mathematics, to their children's mathematics education, and to how the school relates to parents. These measures do not purport to measure cognitive change in children's mathematical performance, and Horne's

analysis of evaluative data concludes that there is no strong link between participation in existing family mathematics programs and children's achievement.

Horne also points out that the impact of such programs needs to be sustained over time. Long-term improvement in parental attitudes and in children's attitudes *and* achievement is more likely to depend on the introduction of other programs and policies into the school over time (cf Horne, 1998, p. 131). Impact on hard-to-reach and ESL parents needs therefore to be related to wider strategies used by schools to communicate with parents, such as the employment of ethnic teacher aides, commitment to family outreach programs, and production of multilingual newsletters.

The *National Numeracy Strategy* (England 1998) has a range of recommendations specifically directed to this end. These point first and foremost to the importance of schools sharing with parents their expectations about what children of different ages and stages of schooling should know and be confident in doing.

Parents need to know how the methods which their children currently follow fit into a larger framework of skills and methods. Parents who have themselves mastered mathematics, readily appreciate that effective teaching does not introduce children to the most sophisticated and efficient strategies from the outset. These strategies need to be built up slowly and grow out of simpler and less formal approaches which are often easier for children to work with and build up the confidence they need before moving to more abstract forms. It is easy for adults to forget the many early steps along the way which precede these more efficient performances. Teachers need to reassure parents about where their children are up to and to suggest ways in which parents can actively support the school's numeracy program. Parents can be greatly helped by documents, such as those developed by the Tasmanian Department of Education, which espouse a broad connected definition of numeracy across school subjects and which exemplify applications of numeracy in daily life and in work.

Regular meetings with parents are important as would be regular inclusion of topics relating to mathematics and numeracy in school newsletters. In this respect, it is very important that consistent messages be given through the media and other public communications. Homework is one area where there is a great deal of potential for schools and parents to work together. Encouraging interest in mathematics in the community at large will also assist in raising consciousness of key issues relating to numeracy and to the teaching of numeracy in schools.

Strategies for the closer involvement of parents in children's numeracy education could include quite simple events such as organising a 'maths trail', displaying examples of children's work, encouraging parents to involve their children in games with a numeracy focus, such as snakes and ladders, simple counting games involving cards, and so on. Schools with a mix of ethnic backgrounds will find many

rich opportunities for using number embedded in games from other cultures. Some strategies to reach parents, and in particular, preschool parents might include: A *'Numeracy Start'* program to help parents assist their children to become confident with simple number and counting skills before they come to school. Much more can be done with young children beyond encouraging them to recite the sequence of counting numbers. Such a program should include a special focus on parents of NESB. It also needs to aim at improving parents' numeracy skills. But most importantly, it should be aimed at supporting parental involvement and interest in children's numeracy development. Early interest and involvement send powerful messages to children about the value of what is being learned, far more so than particular activities considered in isolation.

Implications for policy in Australian schools

Very little research has taken place in this area to date. A range of strategies should be explored nationally to assist parents to support their children's growth in numeracy. Dissemination of successful program and strategies is essential. Informing parents, especially parents of pre-school children, about the importance of numeracy and of ways in which they can support their children's development is one approach that should be investigated. The support of key parent bodies, such as the Australian Council of State School Organisations (ACSSO) and the Australian Parents Council (APC), will be essential in reaching parents. Information kits, videos and parent/school programs, such as a *'Numeracy Start'* program should also be investigated. There is a need for ongoing research and dissemination of overseas research relating to parents' role in their children's numeracy development. Above all, there has to be a change in the perception that it is 'cool' or 'okay' to say 'I'm no good at maths'. It is that realignment of expectations which is at the heart of Australia's national literacy and numeracy goals, with which I introduced this paper.

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