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The Australian Association of Mathematics Teachers Inc.

ABN 76 515 756 909

POST GPO Box 1729, Adelaide SA 5001

PHONE 08 8363 0288

FAX 08 8362 9288

EMAIL office@aamt.edu.au

INTERNET www.aamt.edu.au

CHAPTER 13.

MODELLING PROPORTIONAL THINKING WITH TWOS AND THREES*

PAUL TABART
Department of Education, Tas.

JANE SKALICKY
University of Tasmania
<jane.skalicky@utas.edu.au>

JANE WATSON
University of Tasmania
<jane.watson@utas.edu.au>

Proportional reasoning is a challenging yet central concept for students in the middle grades, and lays an important foundation for the mathematics that is studied later in high school. Proportionality, and the multiplicative relationship that exists between the quantities being represented, is encountered across the mathematics curriculum with contexts and concepts involving fractions, decimals, percents, ratios, similarity, scaling, probability, and linear relationships. Very often the teaching sequence in the mathematics curriculum proceeds as: fractions, decimals, percents, ratios, in that order, with very little attempt at integration. What we describe here is a sequence that evolved across two professional development sessions. Its aim was to create links across models, mathematical content, and examples from social contexts in the media.

Assuming that our teachers were acquainted with the ideas involved, we started with a summary of ways to solve problems using proportional reasoning. After some introductory work with classic juice mixing problems using concrete materials, a summary of the four types of proportional reasoning problems suggested from the research of Lamon (1993) was presented:

- *part–part–whole* where a subset (part) of a whole is compared with its complement (other part) or the whole itself; for example, boys:girls in a class;

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- *associated sets* where two quantities, not ordinarily associated, are related through a problem context or situation; for example, pizzas:children or people:cars;
- *well-known measures* where well-known entities or rates are expressed (e.g., kilometres per hour, dollars per kilogram);
- *stretching and shrinking* where a relationship exists between continuous, rather than discrete, quantities; for example, height or length where the relationship can be scaled up or scaled down.

These problem types were illustrated using missing value, numerical comparison, and qualitative prediction and comparison tasks, to provide teachers with a range of ways to pose proportional reasoning tasks. Posing problems in a variety of formats elicits the use of multiple solution strategies and therefore encourages students' proportional thinking (Cramer & Post, 1993).

As an application of proportional reasoning, the focus was placed on "associated sets." Many associated sets expressed as a ratio are in fact averages, calculated as arithmetic means. Saying there is 1 pizza for every 2 children, does not necessarily mean that every child eats half a pizza each, but that on average if we had 20 children at a party we would need 10 pizzas to feed them all. "Well-known measures" are further examples that demonstrate the important connection between the concepts of proportionality and averages. Saying we drive at "100 km/hour" from Hobart to Launceston does not mean we drive at a constant speed but that in the end we divided the 200 km by the 2 hours it took, in order to get an average speed of 100 km/hr.

One of the examples we chose for teachers to consider was based on a newspaper article from *The Mercury* in Hobart, which lamented the heavy traffic on Hobart's famous Tasman bridge (see Figure 13.1). The claim was made that if the number of people per car could be raised from 1.2 to 1.5, then there would be up to 15 000 fewer cars on the bridge per day (Let someone else drive, 1994; Watson, 1996).

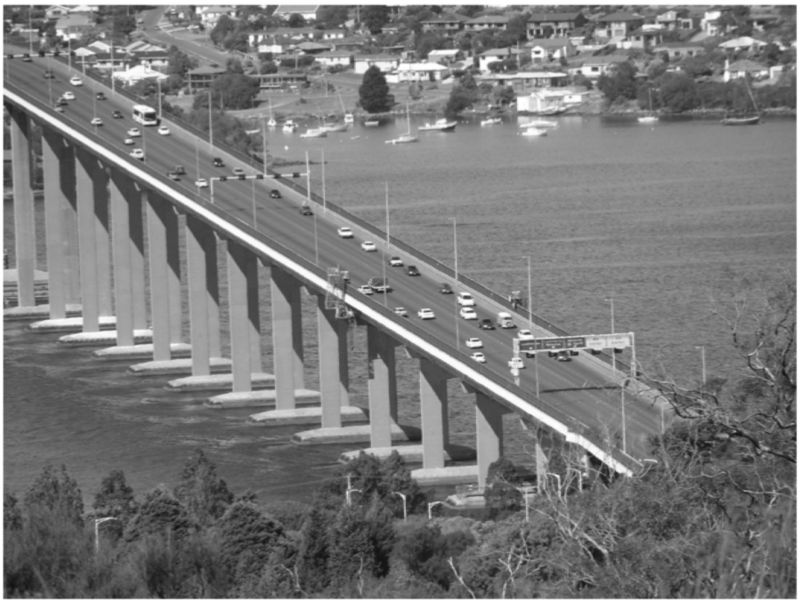

Figure 13.1. The Tasman Bridge, Hobart.

Our point in using this example was to get teachers to consider what the bridge would look like, in terms of cars and people, if the average were actually 1.5 people per car. Teachers were asked to draw various scenarios, compare them, and consider developing a rubric for assessing the task if given to their students. Between professional development sessions, several teachers tried the activity in their classes and two of the suggested scenarios from Grade 6 students are shown in Figure 13.2. Both scenarios represent 6 cars: 9 people but in two different ways. The class group is shown in Figure 13.3 with a display of their work. This context was felt by teachers to be an interesting and perhaps unexpected use of proportional thinking.

One question arose however: Is it possible to have three cars, or in fact any odd number of cars on the bridge if the average is exactly 1.5 people per car? To explore this issue, first the teachers were asked to list some possibilities for the numbers of cars and people in a table. This produced some rather ad hoc lists like in Figure 13.4. Patterns were then established and expanded as shown in Figure 13.5.

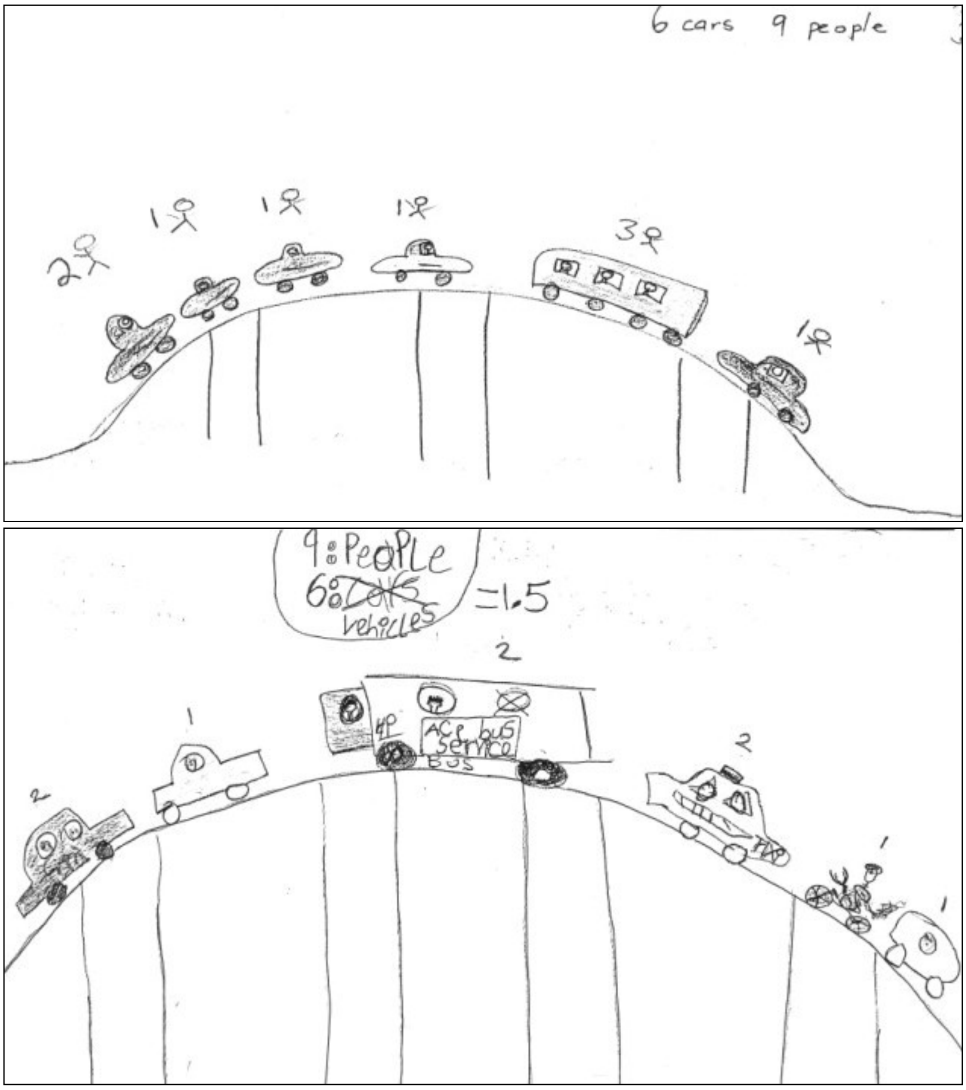

Figure 13.2. Grade 6 students represent “1.5 people per car”.

Figure 13.3. Grade 6 students display their work for “1.5 people per car”.

Cars	People
2	3
4	6
10	15
100	150
6	9

Figure 13.4. Random suggestions for "1.5 people per car".

Cars	People
2	3
4	6
6	9
8	12
10	15
12	18
14	21
16	24

Figure 13.5. Developing a pattern for "1.5 people per car".

This was further reinforced with the use of blue and yellow circular counters. After some initial work with counters in ratios such as 1:3 and 1:5, for various numbers of blue chips (e.g., 1 blue : 3 yellow, 2 blue : 6 yellow, 5 blue : 15 yellow), teachers were asked to model the 3:2 ratio they had discovered for the bridge problem. Figure 13.6 shows a teacher working with this task. They were then asked to represent the ratios in Figure 13.5 with counters. From the observations many teachers concluded that they could not think of a case with an odd number of cars. Why?

Figure 13.6. Modelling 3:2 with coloured counters.

To explore this idea further, teachers were given graph paper and asked to draw a scattergram of the data in Figure 13.5. Patterns in the table were discovered to have a relationship to the pattern of dots on the graph. In fact in plotting the points many teachers were seen to count "two lines across and three lines up" when plotting their points. They were then asked to connect the dots and explore the line. The teachers realised the linear relationship long before drawing the line but when asked to read values for 3, 5, and 7 cars, an "aha" experience could be heard from many: At 1 it is 1.5 people — of course! That was the average! So at each odd number we have a "*half* a person."

This was judged by teachers to be a useful and motivating middle school activity and was extended for some to reinforce the definition of slope of the line. Since

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

we see 3 people "up" for each 2 cars "across" or

$$\text{slope} = \frac{3}{2} = 1.5,$$

again back to the average. For every increase in one car we theoretically increase the number of people by 1.5; but this only makes sense in the real world for every two cars. These ideas are summarised in Figure 13.7.

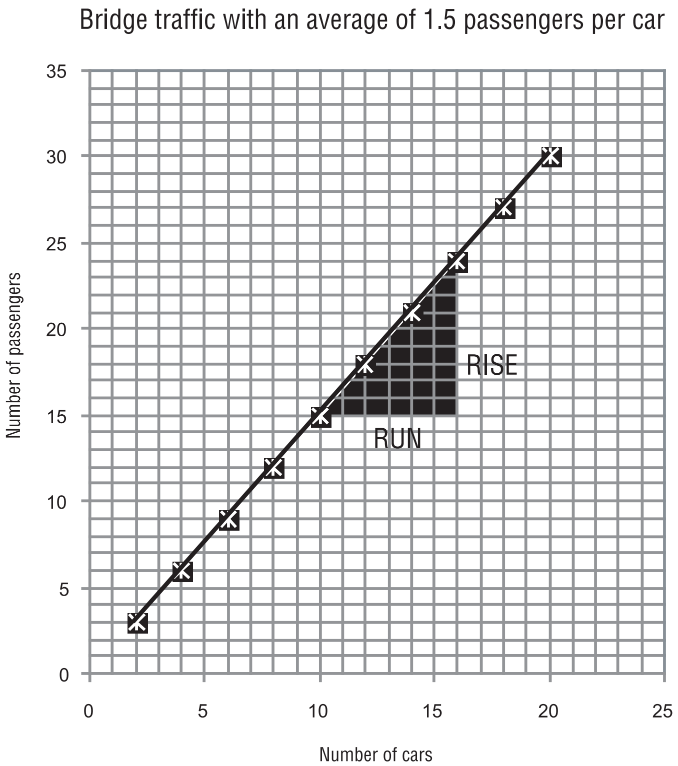

Figure 13.7. Graph of "1.5 people per car".

Further links to algebra can also be made here. All proportional situations like this one can be expressed by the linear equation $y = mx$, where m is the slope and the points on the graph lie on a straight line. In the example of "people per car" the algebraic expression that describes the proportional relationship is therefore:

$$\text{Number of passengers} = 1.5 \times \text{number of cars}$$

It is important across the middle school years to help students make the connections, within the mathematics, which are discussed here. Real-world contexts can provide the opportunity to demonstrate the links from modelling with counters, to tables, to graphical representations, and to algebraic forms of the underlying proportional situation being explored. Although it is unlikely that all of these connections will be covered at the same time, it is essential that teachers, even in the lower middle grades, are aware of the potential to use them to extend their students.

Taking advantage of the numbers involved to transfer understanding to a related concept, percentage increase, another newspaper article from *The Mercury* in Hobart was introduced. Percent is a special type of ratio where the denominator of one ratio pair is always 100. Percent increase or decrease could be classified as a "stretching and shrinking" problem, although it is noted that in this case the relationship being considered is the scaling up of a discrete quantity not a continuous quantity as described by Lamon (1993). The newspaper article described the increase in the number of flies allowed on fishing lines in Tasmania from two to three in percent terms, claiming the increase was 33% (Dally, 1999).

Elsewhere this article has been used to illustrate innumeracy in the press (Watson, 2004) but the point here was to model, with counters or otherwise, the increase in the allowed number of flies and describe it as a percent increase. Then there was the issue of describing how the reporter could perhaps have believed that the increase was 33%, not the 50% modelled by teachers with objects as shown in Figure 13.8. The ease of comparing the language of wholes and parts with this model seemed apparent: 2 flies are the original whole, and 1 extra fly compared to 2 is "half" or "50%" of the whole. Examining a different whole of 3 flies is probably what led the reporter to suggest 33%, but this would only be relevant if the government were reducing the number of flies from 3 to 2, rather than increasing from 2 to 3.

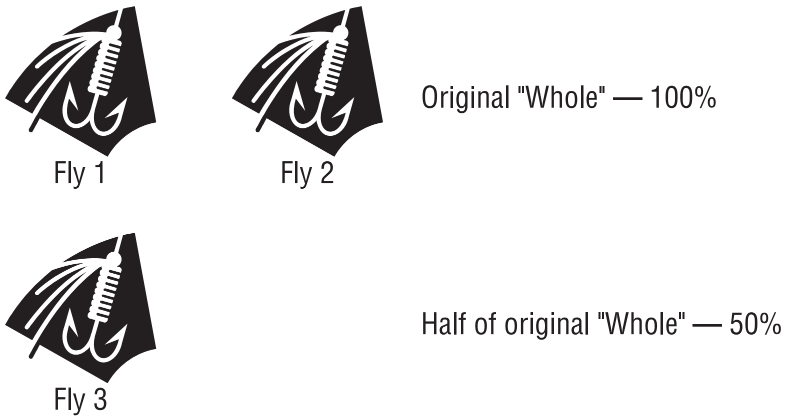

Figure 13.8. Fly fishing per cent model.

What is different about the two scenarios used here? In the first case we are considering different types of quantities: 3 people to 2 cars. In the second case we are comparing quantities of the same type: 2 flies to 3 flies. In the first case, ratios and averages, here involving decimals, are appropriate to describe the relationship of people to cars. In the second case, percents and fractions are appropriate to describe the relationship of the "old" legal number of flies to the "new" legal number of flies. This is why the representations using counters look different. Whereas 5 counters (3 blue and 2 yellow) are needed for the people per car scenario, only 3 counters are needed altogether to describe the fly scenario. Furthermore, it is not necessary for the counters in the fly scenario to be different in colour as the percent increase being considered is concerned with the scaling up of the same entity, in this case, flies. Of course it is also possible to discuss the fly context with a similar ratio:

new number of flies : old number of flies = 3:2 .

This can be linked to the decimal 1.5 but in this case the decimal does not help describe the situation as well as the associated percent, 150%. Here 150% represents the new number of flies in relation to the old number (100%) and again we see the 50% increase. Although the use of counters in modelling is different for the two contexts, much of the formal mathematics is the same.

We admit that combining the ideas of ratio comparison (part-part or associated sets) of different entities (people and cars) and of percent comparison (stretching and shrinking) of the same entity (flies) may be challenging for some. The use of contexts relevant outside the classroom, however, may reinforce differences and similarities and give people (teachers and students) something concrete to recall and therefore contribute to understanding. Recognising connections among mathematical topics, and specifically drawing attention to them, gives students an expectation that the ideas they learn are useful in solving other problems and exploring other concepts.

The use of context also illustrates our belief that quantitative literacy is essential across the curriculum to build links between mathematical concepts and their applications. These opportunities should not be missed.

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