



Pythagoras similarity proof

<http://topdrawer.aamt.edu.au/Geometric-reasoning/Good-teaching/Writing-a-proof/Proving-Pythagoras-theorem/Dissected-proof>

Aim: To prove $c^2 = a^2 + b^2$

Proof: In $\triangle ABC$ and $\triangle ADC$

$\angle A$ is common

$\angle ACB = \angle ADC$ (both 90° given)

$\therefore \triangle ABC \sim \triangle ADC$ (AAA)

$\therefore \frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$ (matching sides of similar triangles)

$$\frac{c}{b} = \frac{a}{CD} = \frac{b}{y}$$

$$\therefore \frac{c}{b} = \frac{b}{y}$$

$$\therefore b^2 = cy$$

In $\triangle ABC$ and $\triangle BDC$

$\angle B$ is common

$\angle ACB = \angle BDC$ (both 90° given)

$\therefore \triangle ABC \sim \triangle CBD$ (AAA)

$\therefore \frac{AB}{CB} = \frac{BC}{BD} = \frac{AC}{CD}$ (matching sides of similar triangles)

$$\frac{c}{a} = \frac{a}{x} = \frac{b}{CD}$$

$$\therefore a^2 = cx$$

$$\begin{aligned}\text{Now } a^2 + b^2 &= cx + cy \\ &= c(x + y) \\ &= c(c) \\ &= c^2\end{aligned}$$

$$\therefore c^2 = a^2 + b^2$$

